NPS55-79-030

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Seasonal Production and Sales Planning
with Limited Shared Tooling
at the Key Operation

by

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December 1979

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Naval Postgraduate School
Monterey, California

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS55-79-030	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Seasonal Production and Sales Planning with Limited Shared Tooling at the Key Operation		5. TYPE OF REPORT & PERIOD COVERED Technical
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) G. G. Brown, A. M. Geoffrion, and G. H. Bradley		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, Ca. 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, Ca. 93940		12. REPORT DATE December 1979
		13. NUMBER OF PAGES 24
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Optimization,
Production Planning,
Integer Optimization,

Network Optimization,
Lagrangean Relaxation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Lagrangean relaxation is applied to a class of very large mixed integer linear programming problems representing seasonal production and sales planning in a situation where limited tooling is available at the key production operation. A successful application to the injection molding industry is described.

This paper addresses production and sales planning in a seasonal industry with a single dominant production operation for which tooling can be shared among parts and is limited in availability.

The specific context of our experience is the production of injection molded plastic pipes and fittings destined for the building and chemical industries. The dominant production operation is injection molding and the tooling consists of mold bases used to adapt the injection molding machines to the molds proper. Mold bases typically require 4-6 calendar months to obtain at a cost which can approach the cost of the molding machine itself so their availability is limited and good utilization is important.

Dominant production operations with limited and possibly shared tooling arise in many other contexts. Likely candidates include production facilities based on casting, molding, stamping, extrusion, or pressing of finished or nearly finished products. Dies and molds and associated adaptive tooling are usually expensive and often designed for use with more than one end product. Machine tooling with elaborate jigs and fixtures constitutes another large area of potential application.

An informal statement of the problem treated is as follows. A facility produces many different parts (products), each by a single operation calling for a specific type of tool and any one of a number of machines compatible with the tool. Machines are aggregated into machine groups and tools into tool types. Production and sales are to be planned for each part over a multiperiod horizon (typically monthly for a full year):

Determine

- how much of each part to <u>produce</u> in each time period
- how much of each part to <u>sell</u> in each time period
- how much of each part to carry forward as <u>inventory</u> from each time period into the next

- a tool/machine assignment schedule specifying, for each time period, the number of days of production of each tool type in conjunction with each compatible machine group

so as to satisfy all necessary constraints

- limited availability of tools in each time period
- limited availability of machines in each time period
- tool/machine compatibility restrictions
- for each part in each time period, sales cannot exceed forecast demand

and so as to satisfy desired managerial policy constraints

- for each part in each time period, sales must exceed a certain fraction of demand stipulated by management
- for each part, the ending inventory at the conclusion of the planning horizon must take on a stipulated value
- no planned backlogging of unfilled demand

in such a manner as to maximize total profits over all parts
for the duration of the planning horizon, calculated according to

- incremental net profit contribution per unit produced
- less variable operating costs associated with production (by tool type and machine group)
- less fixed costs associated with production (by part, for each period with positive production)
- less inventory holding costs.

The problem as stated has elements in common with many familiar dynamic planning and resource allocation problems. It is more detailed than most seasonal planning problems in that discrete fixed costs are included and no aggregation is necessary over parts, yet it stops short of encompassing detailed scheduling because other aggregations are employed (tools → tool types, machines → machine groups, time → time periods). Related production planning and scheduling problems in the molding industry can be found in [3] [6] [7].

A proper mathematical formulation as a mixed integer linear program is given in Sec. 1. The next section presents a solution approach based on a

particularly attractive Lagrangean relaxation and sketches our full scale

computational implementation. Sec. 3 describes computational experience

with the injection molding application mentioned earlier. For this

application, solutions well within 2% of optimum are routinely produced

in about 3 minutes of IBM 370/168 time for mixed integer linear programs

on the order of 12,000 binary variables, 40,000 continuous variables,

and 26,000 constraints.

1. THE MODEL

This section formally defines and discusses the model as a mixed integer linear programming problem.

The formulation makes essential use of the concept of a <u>standard day</u>, which is a part-specific measure of quantity. It is, for a given part, the quantity that would be produced in one calendar day if a tool of the required type were operating normally on any compatible machine.

Notation

Indices

i	indexes parts
j	indexes tool types
k	indexes machine groups
t	indexes time periods, $t = 1,\ldots,T$
$I(j)$	index set of the parts requiring tool type j
$K(j)$	index set of the machine groups compatible with tool type j

Given Data

a_{jt}	days of availability of type j tools during period t
b_{kt}	days of availability of machine group k during period t
c_{jkt}	variable daily operating cost during period t of tool type j on machine group k, for compatible combinations of j and k
d_{it}	demand forecast for part i in period t, in standard days
f_{it}	fixed cost associated with the production of part i in period t
h_{it}	holding cost for one standard day of part i held for the duration of period t
I_{io}	initial inventory in period 1 of part i, in standard days (must be ≥ 0)
I_{iT}	ending inventory desired for part i, in standard days, at the conclusion of the last period (must be ≥ 0)
m_{it}	maximum possible production of part i in period t, in standard days
p_i	profit contribution associated with one standard day's worth of part i, exclusive of the other costs included in the model
α_{it}	minimum fraction of d_{it} which must be satisfied as a matter of marketing policy

Decision Variables

I_{it} planned inventory of part i at the conclusion of period t, in standard days $(1 \leq t < T)$

S_{it} planned sales of part i in period t, in standard days

W_{jkt} planned production days for tool type j on machine group k during period t, for compatible combinations of j and k

X_{it} planned production of part i during period t, in standard days

Y_{it} a binary variable indicating whether or not part i is produced during period t

Mixed Integer Linear Program

$$\text{MAXIMIZE} \atop {I,S,W,X,Y}$$

(1)
$$\sum_i p_i \sum_t X_{it} - \sum_j \sum_k \sum_t c_{jkt} W_{jkt}$$
$$- \sum_i \sum_t h_{it}(I_{i,t-1} + I_{i,t})/2 - \sum_i \sum_t f_{it} Y_{it}$$

subject to

(2) $\sum_{k \in K(j)} W_{jkt} \leq a_{jt}$, all jt

(3) $\sum_{k \in K(j)} W_{jkt} = \sum_{i \in I(j)} X_{it}$, all jt

(4) $\sum_j W_{jkt} \leq b_{kt}$, all kt

(5) $I_{it} = I_{i,t-1} + X_{it} - S_{it}$, all it

(6) $\alpha_{it} d_{it} \leq S_{it} \leq d_{it}$, all it

(7) $0 \leq X_{it} \leq m_{it} Y_{it}$, all it

(8) $W_{jkt} \geq 0$, all jkt

(9) $I_{it} \geq 0$, all it $(1 \leq t < T)$

(10) $Y_{it} = 0 \text{ or } 1$, all it

It is understood that any summations or constraint enumerations involving j and k together will run only over compatible combinations of j and k.

The objective function (1) is essentially the profit over the duration of the planning horizon. It is the profit contribution associated with production over the planning horizon, less: machine operating costs, inventory holding costs (applied to a simple 2-point estimate of the average inventory level of each part in each period), and fixed costs.

Constraints (2) and (4) respectively enforce availability limitations on tools (by type) and machines (by group). Constraint (3), a work balance on tools, relates the X's to the W's. Constraint (5) defines ending inventories in the standard way. Constraint (6) requires the planned sales to be between forecast demand and some specified fraction thereof. Constraint (7) keeps production within possible limits and also forces Y_{it} to be 1 when X_{it} is positive. Constraint (9) specifies that there be no planned backlogging. Constraints (8) and (10) require no comment.

Further Discussion

Some additional comments are appropriate.

1. There can be more than one tool (resp. machine) available of a given type (resp. group). Such census information, along with downtime estimates, determines the a_{jt} (resp. b_{kt}) coefficients.

2. The index sets I $(\cdot)$ must be mutually exclusive and exhaustive, and hence constitute a partition of the part indices. A unique tool type thus is specified for each part. Tooling is common to multiple parts to the extent that these index sets are not singletons.

3. The fixed cost coefficients f_{it} are perhaps best interpreted as surrogates for detailed setup costs. The reason is that f_{it} is incurred when part i is produced in period t irrespective of whether this requires a tool changeover (part i's tool type may be common to the part run previously), and irrespective of whether more than one m must simultaneously make part i in order to achieve the planned production X_{it}; to specify setup costs at this higher level of detail would require a major revision of the model that would transport it from the realm of planning to the realm of detailed scheduling. Yet setup costs cannot be ignored entirely because

this tends to cause some of every part to be produced during every
period, a situation clearly unacceptable from the production
viewpoint. Our solution is to take the f_{it}'s as empirically weighted
average setup costs.

4. Ending inventory level is the only significant terminal condition
   of the model. A plausible choice is to set I_{iT} equal to half the
   desired lot size plus the desired seasonal inventory for part i at
   the time in the seasonal cycle corresponding to the end of period T
   (based on historical operating experience, insights obtained
   previously with the help of the model, and managerial judgment).

5. The maximum possible production m_{it} is the smaller of two limits:
   the physical limit imposed by full utilization of all available
   tooling and machines, and the limit on the amount of production
   that could be absorbed considering the total demand over the planning
   horizon, specified ending inventory, and current inventory.

6. The profit coefficient p_i is applied to $\Sigma_t X_{it}$ instead of to
   $\Sigma_t S_{it}$. The rationale for this is that everything made will be sold
   sooner or later; applying p_i as indicated avoids the need to value
   initial inventory I_{io} or ending inventory I_{iT}.

7. The rationale for the policy parameters α_{it} is that demand levels
   for different parts may be interdependent: if scarce production
   resources are allocated only to the most profitable parts, thin
   product lines and spotty product availability may displease
   customers and result in lower market share for the profitable items.

Size

For the practical application at hand, problem (1) - (10) has approximately .

   40,000 continuous variables (I,S,W,X)

   12,000 integer variables (Y)

   26,000 constraints of type (2),(3),(4),(5),(7).

Problems of this magnitude are generally considered to be far beyond the current state-of-the-art of general mixed integer linear programming.

2. SOLUTION BY LAGRANGEAN RELAXATION

Lagrangean relaxation [4] [5] [8] with respect to (3) is an attractive way to generate upper bounds on the optimal value of (1) - (10). The Lagrangean subproblem separates into as many independent simple transportation problems in the W variables as there are time periods, and as many independent dynamic single-item lot-size problems as there are parts. The original monolith is thereby decomposed into manageable fragments. A good choice for the Lagrangean variables can be obtained efficiently by solving the conventional LP relaxation of (1) - (10), which is equivalent to a single pure network problem. Moreover, Lagrangean relaxation with respect to (3) does not satisfy the Integrality Property defined in [5] and hence is likely to be an improvement over the conventional LP relaxation.

These observations, explained in detail below, are the basis of a solution procedure that has proven to be quite effective.

One can build a branch-and-bound procedure around this Lagrangean relaxation, but it has not proven necessary to do so for the industrial application which stimulated this work. It has been sufficient to generate a feasible solution to (1) - (10) based on the Lagrangean solution. The objective value of this solution has unfailingly been sufficiently close to the upper bound from Lagrangean relaxation that no further refinement has been needed.

A formal description of the solution procedure is now presented.

Step 1   Solve the usual linear programming relaxation of (1) - (10) via the equivalent capacitated network formulation. Denote the associated dual variables for (3) by $\overline{\lambda}_{jt}$.

Step 2   Form the Lagrangean relaxation of (1) - (10) with respect to (3)

using $\overline{\lambda}$. Separate it into independent subproblems in W for each t

and, for each i, in the remaining variables.

Solve each of the Lagrangean subproblems by specialized algorithms.

Denote the combined optimal solution to the full Lagrangean

relaxation by $(I^{o}, S^{o}, W^{o}, X^{o}, Y^{o})$ and its optimal value by UB.

Step 3   Solve (1) - (10) with Y set "elastically" to Y^{o}; that is,

(10) is relaxed to $0 \leq Y_{it} \leq 1$ with f_{it} set to 0 if $Y^{o}_{it} = 1$ a

augmented by a large positive constant if $Y^{o}_{it} = 0$. Denote the

optimal solution to this problem by (I', S', W', X', Y'). Let Y" be Y'

with all fractional components rounded up to unity. Form the revised

solution (I', S', W', X', Y''), and denote its objective value under (1)

as LB. This solution is feasible in (1) - (10) and is within

UB-LB of being optimal. Stop.

Step 1 yields an equivalent capacitated network problem because, when

(10) is relaxed to $0 \leq Y_{it} \leq 1$ for all i t, the relation $Y_{it} = X_{it}/m_{it}$ must

hold at optimality for all i t. Upon elimination of Y, (7) becomes redundant;

the relaxed version of (10) becomes

(10)' $0 \leq X_{it} \leq m_{it}$, all i t

and (1) can be rewritten as

(1)'   MAXIMIZE $\sum_{i} \sum_{t} \pi_{it} X_{it} - \sum_{j} \sum_{k} \sum_{t} c_{jkt} W_{jkt} - \sum_{i} \sum_{t=1}^{T-1} h'_{it} I_{it} - H$
        I,S,W,X

where

(11) $\pi_{it} \triangleq P_i - \dfrac{f_{it}}{m_{it}}$ for all i t

(12) $h'_{it} \triangleq \dfrac{1}{2} (h_{it} + h_{i,t+1})$ for all i and t=1,...,T-1

(13) $H \triangleq \dfrac{1}{2} \sum_{i} (h_{i1} I_{io} + h_{iT} I_{iT})$

It is easy to see that the resulting linear programming problem can be

formulated as a minimum cost capacitated network flow problem.

See Figure 1 for an example with 3 parts, 2 tool types, 3 machine groups, 3 time periods, $I(1) = \{1\}$, $I(2) = \{2,3\}$, $K(1) = \{1,2\}$, and $K(2) = \{2,3\}$. The notational conventions followed in Figure 1 are: the term of $(1)'$ corresponding to each arc is written over the arc (omission means that the unit flow cost is 0), the upper capacity limit of each arc is written under it (omission implies infinite capacity), and the constraint on the net outflow of each node is written under it (omission implies = 0, or strict conservation). The curved arcs between the part nodes are not annotated for lack of room; the typical arc is:

period t   period t+1

Several variants of the network formulation pictured in Figure 1 are possible.

The Lagrangean relaxation of Step 2 is composed of the following independent subproblems: for each t,

(R^t) $\underset{W_{..t}}{\text{MINIMIZE}} \quad \sum_j \sum_{k \in K(j)} (c_{jkt} - \bar{\lambda}_{jt}) W_{jkt}$

subject to (2),(4), and (8) for fixed t

and, for each i,

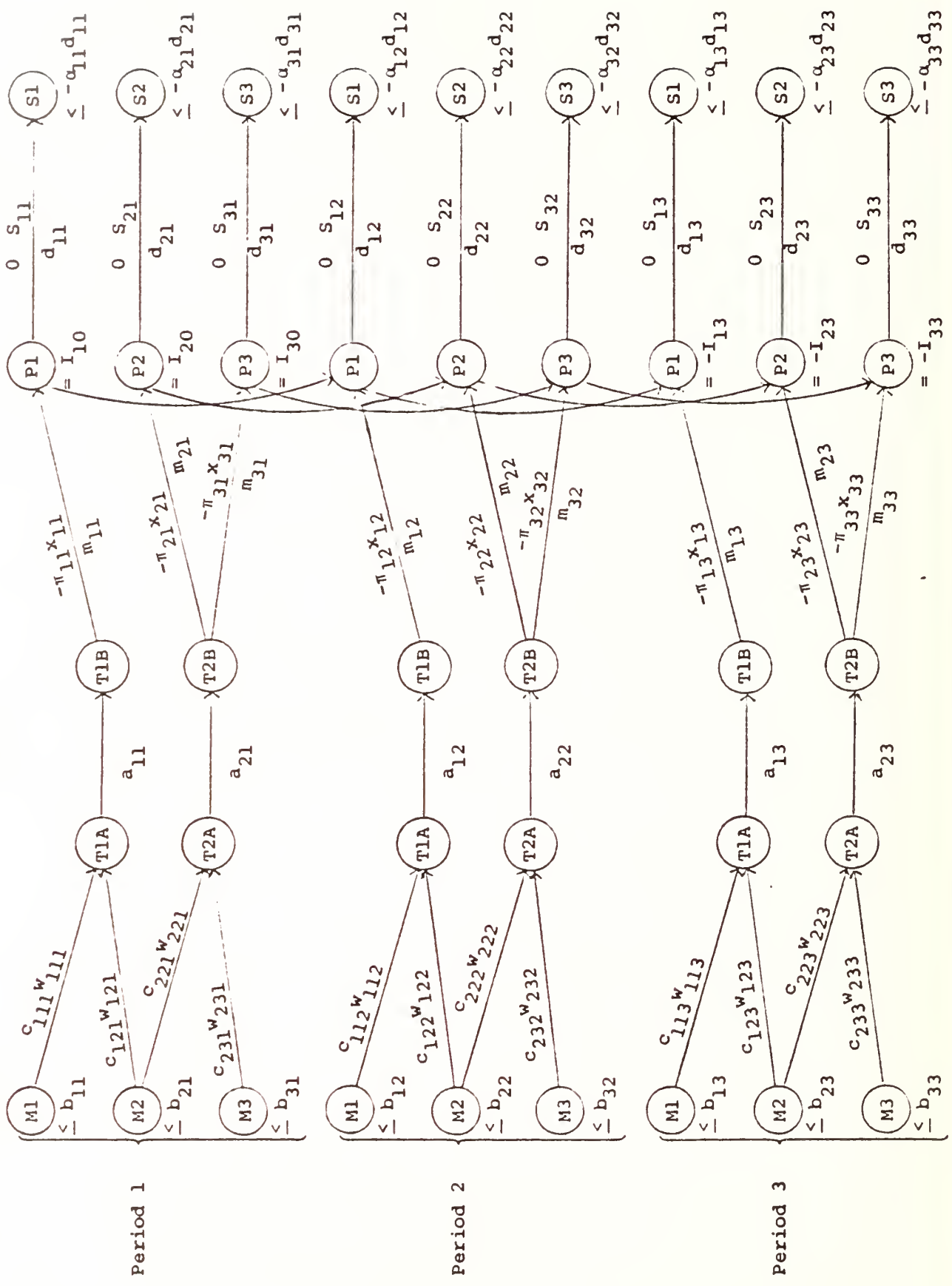(R_i) $\underset{\substack{I_{i.}, S_{i.}, \\ X_{i.}, Y_{i.}}}{\text{MINIMIZE}} \quad \sum_t (\bar{\lambda}_{j(i)t} - p_i) X_{it} + \sum_{t=1}^{T-1} h'_{it} I_{it} + \sum_t f_{it} Y_{it}$

subject to (5),(6),(7),(9),(10) for fixed i

MACHINES   TOOLS   PARTS

Period 1

$M1 \leq b_{11}$ — $c_{111}w_{111}$ → $T1A$ — a_{11} → $T1B$ — $-\pi_{11}x_{11}$, m_{11} → $P1 = I_{10}$ — $0 \; s_{11}$, d_{11} → $S1 \leq -\alpha_{11}d_{11}$

$M2 \leq b_{21}$ — $c_{121}w_{121}$, $c_{221}w_{221}$ → $T2A$ — a_{21} → $T2B$ — $-\pi_{21}x_{21}$, m_{21} → $P2 = I_{20}$ — $0 \; s_{21}$, d_{21} → $S2 \leq -\alpha_{21}d_{21}$

$M3 \leq b_{31}$ — $c_{231}w_{231}$ — $-\pi_{31}x_{31}$, m_{31} → $P3 = I_{30}$ — $0 \; s_{31}$, d_{31} → $S3 \leq -\alpha_{31}d_{31}$

Period 2

$M1 \leq b_{12}$ — $c_{112}w_{112}$ → $T1A$ — a_{12} → $T1B$ — $-\pi_{12}x_{12}$, m_{12} → $P1$ — $0 \; s_{12}$, d_{12} → $S1 \leq -\alpha_{12}d_{12}$

$M2 \leq b_{22}$ — $c_{122}w_{122}$, $c_{222}w_{222}$ → $T2A$ — a_{22} → $T2B$ — $-\pi_{22}x_{22}$, m_{22} → $P2$ — $0 \; s_{22}$, d_{22} → $S2 \leq -\alpha_{22}d_{22}$

$M3 \leq b_{32}$ — $c_{232}w_{232}$ — $-\pi_{32}x_{32}$, m_{32} → $P3$ — $0 \; s_{32}$, d_{32} → $S3 \leq -\alpha_{32}d_{32}$

Period 3

$M1 \leq b_{13}$ — $c_{113}w_{113}$ → $T1A$ — a_{13} → $T1B$ — $-\pi_{13}x_{13}$, m_{13} → $P1 = -I_{13}$ — $0 \; s_{13}$, d_{13} → $S1 \leq -\alpha_{13}d_{13}$

$M2 \leq b_{23}$ — $c_{123}w_{123}$, $c_{223}w_{223}$ → $T2A$ — a_{23} → $T2B$ — $-\pi_{23}x_{23}$, m_{23} → $P2 = -I_{23}$ — $0 \; s_{23}$, d_{23} → $S2 \leq -\alpha_{23}d_{23}$

$M3 \leq b_{33}$ — $c_{233}w_{233}$ — $-\pi_{33}x_{33}$, m_{33} → $P3 = -I_{33}$ — $0 \; s_{33}$, d_{33} → $S3 \leq -\alpha_{33}d_{33}$

where $j(i)$ is the index of the tool type required by part i. Using an obvious notation, the optimal value of the full Lagrangean relaxation of Step 2 is

(14) $\text{UB} \stackrel{\Delta}{=} - \sum_t v(R^t) - \sum_i v(R_i) - H.$

Figures 2 and 3 portray (R^t) and (R_i) for the example illustrated in Figure 1. Notational conventions are the same as before except for the arcs leaving node Q_i in Figure 3: these are dashed to indicate that they are "fixed charge" arcs, with the amount of fixed charge incurred by their use given as the first of the two annotations written over the arcs.

Problem (R^t) can be converted easily to a simple transportation problem. However, it can be shown using LP duality theory that the W-part of the optimal solution found at Step 1 is necessarily optimal also in these sub-problems. No work at all need be performed in connection with these subproblems!

Problem (R_i) has as many fixed charge arcs as there are time periods. Its special structure invites the development of a specialized solution procedure (e.g., [1] treats a closely related class of dynamic lot-size problems which is a special case of (R_i)).

Step 3 yields a problem virtually identical in form to that of Step 1. It can be solved efficiently in the same manner.
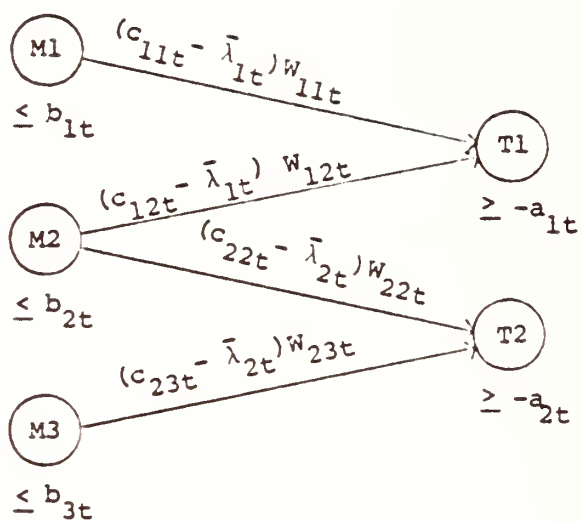
FIGURE 2: SAMPLE EQUIVALENT NETWORK FOR (R^t)
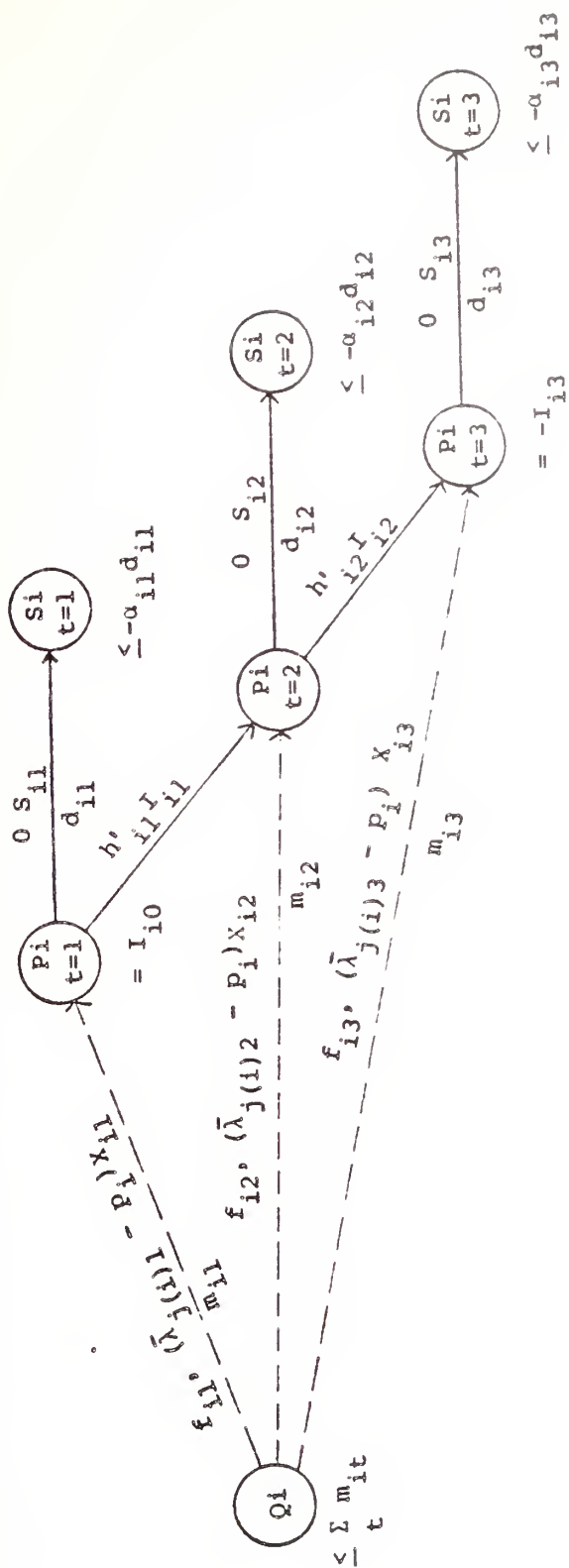
FIGURE 3: SAMPLE EQUIVALENT FIXED CHARGE NETWORK FOR (R_i)

3. APPLICATION AND COMPUTATIONAL RESULTS

The model and computational procedure described above have been under
development and application for more than two years at both plants of R & G
Sloane Manufacturing Company of Sun Valley, California. The following identi-
fications and specializations are appropriate.

General Model	Molding Application (main plant)
parts (i)	the top 1000 injection molded fittings (about 92% of all sales volume)
tool types (j)	about 90 types of interchangeable mold bases (total mold base census about 130)
machine groups (k)	about 15 groups of interchangeable injection molding machines (total machine census about 60)
tool/machine compatibility	about 480 jk combinations permissible
time periods (t)	typically the next 12 months
c_{jkt}, f_{it}, α_{it}	taken as independent of t

The problem faced by R & G Sloane is a strongly seasonal one; with the
bulk of the company's business accounted for by residential plumbing products,
demand peaks along with residential construction in the summer months.
Since the peak season demand rate exceeds the available capacity of mold
bases and machines, constraints (2) and (4) tend to be binding at that time
of year (typically, about 20% of the mold base constraints and 80% of the
machine constraints are binding in at least three months). Typical relative

magnitudes of the major cost categories associated with an optimal solution
are:

fixed costs	14.3
inventory carrying	16.9
variable operating	68.8
	100.0

Unfilled demand in most optimal solutions occurs for 2 or 3% of all parts.

Computational Implementation

A full scale computational implementation has been carried out for this
application. The computer programs are in three modules:

1. data extraction and data base definition
2. problem preprocessing and diagnosis
3. optimization and report writing.

Data extraction primarily involves conversion of current production,
marketing, and inventory control operating data to the form required by the
model. The data base is organized and generated in sections:

- problem parameters and conditions
- machine group descriptions
- mold bases and their machine compatibility
- part descriptions and demand forecasts.

Preprocessing identifies structural and mathematical inconsistencies
in the problem posed, and assists in preliminary diagnosis of critical
shortages in equipment availability.

The optimization module solves the capacitated pure networks presented in Steps 1 and 3 with a GNET variant (XNET/Depth)[2]; an advanced starting solution is used which assumes high equipment utilization. The fixed charge problems (R_i) are solved in Step 2 with a highly specialized fixed order enumeration algorithm employing GNET/Depth; key elements of this procedure identify and exploit dominant problem features such as mandated maximum production, and permit parametric relaxation of the full enumeration for prohibitively long solution sequences.

The solution reports are presented at several levels of aggregation so as to facilitate managerial interpretation. They display all detailed solution features, estimated opportunity costs for critical mold bases and machines, and an overall analysis of profitability, turnover, and customer service.

Computational Results

Approximately 30 runs have been made during the last year. Computational performance has exhibited a high degree of run-to-run stability in terms of the quality of solutions produced and the amount of computer resources expended.

Table 1 summarizes several aspects of performance for a recent typical run of the optimization module. With report writing time included, the total CPU time for this run was 165.5 seconds. The main storage requirement was about one megabyte. Notice that the bound produced by the Lagrangean relaxation is significantly better than the ordinary linear programming relaxation bound. Notice also that the time in Step 2 is smaller than what one might expect; the 12-period fixed charge problems were solved in an average of only .027 seconds each (for comparison, the typical time quoted in [1] for a proper subclass of

	Pivots	IBM 370/168 CPU Seconds	Normalized Objective Value
Step 1			
(LP Relaxation)	37,933	55.0	103.2
Step 2			
(Lagrangean Relaxation)	376,241	25.7	101.6
Step 3			
(Generate Feasible Solution)	36,954	54.2	100.0

TABLE 1

Typical Computational Performance
(953 parts, 92 tool types, 16 machine groups)

of these problems of the same size was 0.25 seconds on an IBM 370/158).
For this run, 142 (resp. 10) of the 11,436 binary Y variables changed
from value 0 (resp. 1) in Step 2 to value 1 (resp. 0) in Step 3. This shows
that the solution to the Lagrangean relaxation of Step 2 required but minor
adjustment with respect to the fixed change arcs in order to yield the good
feasible solution of Step 3.

The pre-optimization modules required 10 seconds for the run reported
in Table 1.

More generally, our experience has been that optimization CPU time for
similar sized problems seldom varies more than ± 10%. Computing time is
very nearly proportional to the total number of parts. The final optimality
tolerance (which was 1.6% in the Table 1 run) tends to become tighter
the more tightly capacitated tool and machine availability is; tolerances in

the vicinity of 2/10 of 1% are commonly observed in the most tightly constrained situations. In no case has the tolerance ever exceeded 2%.

4. CONCLUSION

This paper has demonstrated the practical applicability of a procedure based on Lagrangean relaxation to a significant class of integrated production and sales planning models. The particular way in which this procedure is designed thoroughly exploits the recent major advances made for minimum cost network flow problems. Provably good solutions are routinely being obtained in modest computing time to mixed integer linear programs of a size far beyond the capabilities of existing general-purpose mathematical programming systems.

The system is used regularly at R & G Sloane Manufacturing company for production scheduling in the sense that day-to-day scheduling is still performed manually but with the benefit of the system's guidance and predictions of bottlenecks in the future. The integrated nature of the model has made the system valuable as a focal point for coordinating planning activities among the key functional areas of the firm: inventory control, finance, marketing, and production operations. Two specific illustrations are the evaluation of major capital expenditure and interplant equipment transfer opportunities.

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